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MATDIP301

Third Semester B.E. Degree Examination, May/June 2010
Advanced Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: *Answer any FIVE full questions.*

1 a. Express the complex number $\frac{(1+i)(1+3i)}{1+5i}$ in the form $x + iy$. (06 Marks)

b. Prove that $(1+i)^n + (1-i)^n = 2^{\frac{n}{2}+1}\cos\left(\frac{n\pi}{4}\right)$. (07 Marks)

c. Expand $\cos^8\theta$ in a series of cosines multiples of θ. (07 Marks)

2 a. Find the n^{th} derivative of $e^{ax}\sin(bx + c)$. (06 Marks)

b. If $y = a\cos(\log x) + b\sin(\log x)$, prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. (07 Marks)

c. Find the n^{th} derivative of $\frac{x}{(x-1)(2x+3)}$. (07 Marks)

3 a. State Taylor's theorem and expand the polynomial $2x^3 + 7x^2 + x - 6$ in powers of $(x - 1)$. (06 Marks)

b. Expand $\tan x$ in ascending powers of x using MacLaurin's theorem upto the term containing x^4. (07 Marks)

c. If $Z = \frac{x^2+y^2}{x+y}$ prove that $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$. (07 Marks)

4 a. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$. (06 Marks)

b. If $u = f(x, y)$ where $x = r\cos\theta$ and $y = r\sin\theta$, prove that $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$. (07 Marks)

c. If $u = x^2 - 2y$, $v = x + y + z$ and $w = x - 2y + 3z$, find the value of $J\left(\frac{u, v, w}{x, y, z}\right)$. (07 Marks)

5 a. Obtain the reduction formula for $\int \sin^m x \cos^n x\, dx$. (06 Marks)

b. Evaluate $\int_0^a \frac{x^7}{\sqrt{a^2 - x^2}}\, dx$. (07 Marks)

c. Evaluate $\int_0^1 \int_0^x e^{\left(\frac{x}{x}\right)}\, dy\, dx$. (07 Marks)

6 a. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz \, dz \, dy \, dx$. (06 Marks)

b. Prove that $\beta(m,n) = \frac{\lfloor m \rfloor \lfloor n}{\lfloor m+n}$. (07 Marks)

c. Show that $\int_0^{\pi/2} \sqrt{\sin\theta} \, d\theta \times \int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} = \pi$. (07 Marks)

7 a. Solve $3 e^x \tan y \, dx + (1 - e^x) \sec^2 y \, dy = 0$. (06 Marks)

b. Solve $x^2 y \, dx = (x^3 + y^3) \, dy$. (07 Marks)

c. Solve $x \frac{dy}{dx} + y = x^3 y^6$. (07 Marks)

8 a. Solve $\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2 + 2x + 4$. (06 Marks)

b. Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 4\cos^2 x$. (07 Marks)

c. Solve $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = e^{-x} + \sin 2x$. (07 Marks)

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